Function Spaces 13

In memory of Henryk Hudzik and Julian Musielak

8-12 July 2024

Poznań

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About

The International Conferences on Function Spaces have a 38-year tradition. The previous conferences were held in:

- Poznań (1986, 1989, 1992, 1998, 2003, 2012);
- Zielona Góra (1995, 2015);
- Wrocław (2001);
- Będlewo - Banach Conference Center (2006);
- Kraków (2009,2018);
- Poznań (2021) - the conference has been postponed due to COVID-19.

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Timetable

CT: Contributed Talk, IS: Invited Speaker.

Monday, 8 of July

8:00–8:45	**Registration**	
8:45–9:00	**Welcome remarks**	
9:00–9:50	IS	**Invited speaker lecture**
10:00–10:50	IS	**Invited speaker lecture**
11:00–11:30	**Coffee**	
11:30–11:55	CT	**Contributed Talk**
12:00–12:25	CT	**Contributed Talk**
12:30–12:55	CT	**Contributed Talk**
13:15–14:30	**Lunch**	
14:30–15:20	IS	**Invited speaker lecture**
15:30–16:20	IS	**Invited speaker lecture**
16:30–17:00	**Coffee**	
17:00–17:25	CT	**Contributed Talk**
17:30–17:55	CT	**Contributed Talk**
18:00–18:25	CT	**Contributed Talk**
18:30–18:55	CT	**Contributed Talk**

Tuesday, 9 of July

9:00–9:50	IS	**Invited speaker lecture**
10:00–10:50	IS	**Invited speaker lecture**
11:00–11:30	**Coffee**	
11:30–11:55	CT	**Contributed Talk**
12:00–12:25	CT	**Contributed Talk**
12:30–12:55	CT	**Contributed Talk**
13:15–14:30	**Lunch**	
14:30–15:20	IS	**Invited speaker lecture**
15:30–16:20	IS	**Invited speaker lecture**
16:30–17:00	**Coffee**	
17:00–17:25	CT	**Contributed Talk**
17:30–17:55	CT	**Contributed Talk**
18:00–18:25	CT	**Contributed Talk**
18:30–18:55	CT	**Contributed Talk**

Timetable

Wednesday, 10 of July

9:00–9:50	IS	**Invited speaker lecture**
10:00–10:50	IS	**Invited speaker lecture**
11:00–11:30		**Coffee**
11:30–11:55	CT	**Contributed Talk**
12:00–12:25	CT	**Contributed Talk**
12:30–12:55	CT	**Contributed Talk**
13:15–14:30		**Lunch**
16:00–18:00		**Excursion**
18:00		**Conference Dinner**

Thursday, 11 of July

9:00–9:50	IS	**Invited speaker lecture**
10:00–10:50	IS	**Invited speaker lecture**
11:00–11:30		**Coffee**
11:30–11:55	CT	**Contributed Talk**
12:00–12:25	CT	**Contributed Talk**
12:30–12:55	CT	**Contributed Talk**
13:15–14:30		**Lunch**
14:30–15:20	IS	**Invited speaker lecture**
15:30–15:55	CT	**Contributed Talk**
16:00–16:25	CT	**Contributed Talk**
16:30–17:00		**Coffee**
17:00–17:25	CT	**Contributed Talk**
17:30–17:55	CT	**Contributed Talk**
18:00–18:25	CT	**Contributed Talk**
18:30–18:55	CT	**Contributed Talk**

Timetable

Friday, 12 of July

9:00–9:50	IS	**Invited speaker lecture**
10:00–10:50	IS	**Invited speaker lecture**
11:00–11:30	**Coffee**	
11:30–11:55	CT	**Contributed Talk**
12:00–12:25	CT	**Contributed Talk**
12:30–12:55	CT	**Contributed Talk**
13:15–14:30	**Lunch**	

List of Abstracts – Invited Speakers

Factorization of compact operators, weighted backward shifts and compact H-operators

Asuman Güven Aksoy IS

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Ideals of continuous linear operators factoring compactly through ℓ^p $(1 \leq p < \infty)$ is familiar to many (see [3], [4]). In this talk, even tough hypercyclic operators are not compact, considering operator ideals generated by weighted backward shifts, we obtain a factorization theorem for them through ℓ^p [2]. We also present some properties of the approximation spaces of certain operators called H-operators, which generalize the concept of self-adjoint to Banach spaces [1].

References

[1] Asuman G. Aksoy, Daniel A. Thiong *Approximation spaces for H-operators*, will appear in Involve in 2024. ArXiv: 2306.03633v1.

[2] Asuman G. Aksoy, Yunied Puig *Ideals of hypercyclic operators that factor through ℓ^p* Proc. Amer. Math. Soc. 150, (2022), no.2, 691-700.

[3] J. H. Fourie, *Injective and surjective hulls of classical p-compact operators with applications to unconditionally p-compact operators*, Studia Math., 240, (2018), 141-159.

[4] T. Terzioğlu, *A characterization of compact mappings*, Arch. Math. 22, (1971) 26-28.

List of Abstracts – Invited Speakers

On some generalizations of Ascoli-Arzelà theorem

Diana Caponetti **IS**

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To apply the theory of measure of noncompactness is useful to calculate or estimate the class theorem, the problem has been widely considered in the literature. Our aim is to give estimates, which in some cases reduce to precise formulas, of the Kuratowski and the Hausdorff measures in very general Banach spaces of vector-valued functions. Precisely, in spaces of vector-valued bounded functions and of vector-valued bounded differentiable functions. To this end, we use a quantitative characteristic modeled on a new equicontinuity-type concept and classical quantitative characteristics related to pointwise relative compactness. We obtain new regular measures of noncompactness in the spaces taken into consideration. The results are extended to obtain quantitative versions of theorems about compactness in groups of group-valued mappings, endowed with a topology which generalizes the topology of convergence in measure. The Ascoli-Arzelà theorem is generalized to a wide class of function spaces.

References

[1] D. Caponetti, A. Trombetta, G. Trombetta, *Compactness in groups of group-valued mappings*, Mathematics, 10(21), 3973, 2022.

[2] D. Caponetti, A. Trombetta, G. Trombetta, *Regular measures of noncompactness and Ascoli-Arzelà type compactness criteria in spaces of vector-valued functions*, Banach J. Math. Anal. 17, 3(48), 2023.

(Isometries on) Spaces of Lipschitz functions and their preduals

Marek Cúth IS

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Given a metric space (M, d) with a distinguished point $0_M \in M$ we consider the Banach space

$$\mathrm{Lip}_0(M) := \{f : M \to \mathbb{R} \colon f \text{ is Lipschitz and } f(0_M) = 0\}$$

endowed with the norm

$$\|f\| := \sup\{\tfrac{|f(x)-f(y)|}{d(x,y)} : x \neq y \in M\}.$$

There is also a natural (and nowadays quite widely studied) predual of $\mathrm{Lip}_0(M)$ known under the names Lipsphitz-free space, Arens-Eells space, or transportation cost space.

The talk will be divided into two parts. In the first part I will try to survey some of the known Banach space properties of the space $\mathrm{Lip}_0(M)$ and its predual, and also mention some open problems.

In the second part of the talk, I will mention results from our recent paper [1], where we study the structure of linear surjective isometries of the space $\mathrm{Lip}_0(M)$ and its predual. For example, we prove that for quite a surprisingly rich class of metric spaces (including all 3-connnected graphs or non-abelian Carnot groups with horizontally strictly convex norms) every surjective linear isometry of the predual of $\mathrm{Lip}_0(M)$ is generated by a surjective dilation (i.e. rescaled isometry) of the metric space itself.

References

[1] M. Cúth, M. Doucha, T. Titkos, Isometries of Lipschitz-free Banach spaces, preprint available at arxiv.org

List of Abstracts – Invited Speakers

Projection constants for spaces of multivariate polynomials

Andreas Defant

IS

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We discuss several aspects of an ongoing project with D. Galicer, M. Mansilla, M. Mastyło, and S. Muro.

The general problem we address is to develop new methods within the study of projection constants of Banach spaces of multivariate polynomials. The relative projection constant $\boldsymbol{\lambda}(X, Y)$ of a subspace X of a Banach space Y is the smallest norm among all possible projections from Y onto X, and the (absolute) projection constant $\boldsymbol{\lambda}(X)$ is the supremum of all relative projection constants of X taken with respect to all possible super spaces Y. This is one of the most significant notions of modern Banach space theory. We sketch a few abstract ideas which allow to handle in a unified way a wide variety of Banach spaces of polynomials, and emphasize on polynomials on Boolean cubes, Dirichlet polynomials on the complex plane, and polynomials on euclidean spheres.

Semidefinite programming lending three hands to approximation theorists

Simon Foucart

IS

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In this talk, modern optimization techniques are publicized as fitting computational tools to attack several extremal problems from Approximation Theory which had reached their limitations based on purely analytical approaches. Three such problems are showcased: the first problem—minimal projections—involves minimization over measures and exploits the moment method; the second problem—constrained approximation—involves minimization over polynomials and exploits the sum-of-squares method; and the third problem—optimal recovery from inaccurate observations—is highly relevant in Data Science and exploits the S-procedure. In each of these problems, one ends up having to solve semidefinite programs.

Generalised Morrey smoothness spaces

Dorothee D. Haroske

IS

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In the recent past, smoothness spaces of Besov or Triebel-Lizorkin type, built upon Morrey spaces $\mathcal{M}_{u,p}(\mathbb{R}^d)$, $0 < p \leq u < \infty$, have been studied intensively. These scales $\mathcal{N}^s_{u,p,q}(\mathbb{R}^d)$ and $\mathcal{E}^s_{u,p,q}(\mathbb{R}^d)$, $s \in \mathbb{R}$, $0 < q \leq \infty$, have become popular in connection with applications in PDE, but are also interesting for their own sake. They generalise the well-known scales of Besov and Triebel-Lizorkin spaces, $B^s_{p,q}(\mathbb{R}^d)$ and $F^s_{p,q}(\mathbb{R}^d)$, since $\mathcal{M}_{p,p}(\mathbb{R}^d) = L_p(\mathbb{R}^d)$, $0 < p < \infty$. Following a similar intention, the Besov-type and Triebel-Lizorkin-type spaces $B^{s,\tau}_{p,q}(\mathbb{R}^d)$ and $F^{s,\tau}_{p,q}(\mathbb{R}^d)$, $\tau \geq 0$, were introduced and systematically studied in the last years.

Now we follow an idea of Mizuhara and Nakai who introduced in the beginning of the 1990's generalised Morrey spaces $\mathcal{M}_{\varphi,p}(\mathbb{R}^d)$, where $\varphi : (0,\infty) \to (0,\infty)$ stands for a function belonging to a so-called $\mathcal{G}_p$ class, $0 < p < \infty$, and the special setting $\varphi(t) \sim t^{d/u}$, $0 < t < \infty$, covers the case $\mathcal{M}_{u,p}(\mathbb{R}^d)$. In a parallel way one can generalise the spaces $B^{s,\tau}_{p,q}(\mathbb{R}^d)$ and $F^{s,\tau}_{p,q}(\mathbb{R}^d)$ with the help of such a function φ.

We studied basic properties of these new scales of spaces as well as embeddings, decompositions, and local singularity behaviour of distributions in such spaces. One might think, at first glance, that replacing one index (u or τ) by some function $\varphi \in \mathcal{G}_p$ will presumably make the situation more complicated, rather than more transparent and comprehensible. And we admit, that this assumption is surely true in view of the proofs and their technicalities. However, looking at our first results, it turned out that some peculiarities of our recent findings can now be much better classified in the new setting. Roughly speaking, the number of cases to be distinguished when studying embeddings, growth envelopes etc., is reduced to some qualitative limit behaviour of the function φ. We give some survey of our recent results in this direction.

This is joint work with Zhen Liu (Jena), Susana Moura (Coimbra), and Leszek Skrzypczak (Poznań).

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Density results and trace operator for weighted Dirichlet and Sobolev spaces defined on the half-line and applications

Agnieszka Kałamajska IS

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We give an analytic description for the completion of $C_0^\infty(\mathbf{R}_+)$ in weighted Dirichlet space

$D^{1,p}(\mathbf{R}_+,\omega) = \{u : \mathbf{R}_+ \to \mathbf{R} : u$ is locally absolutely continuous on $\mathbf{R}_+ \text{and } \|u^{'}\|_{L^p(\mathbf{R}_+,\omega)} < \infty\}$,

for given continuous positive weight ω defined on $\mathbf{R}_+$, where $1 < p < \infty$. The conditions are described in terms of the modified variants of the B_p conditions due to Kufner and Opic from 1984, which in our approach are focusing on the integrability of $\omega^{-p/(p-1)}$ near zero or near infinity. We also propose applications of our results to: obtaining new variants of Hardy inequality, interpretation of boundary value problems in ODEs defined on the half-line with solutions in $D^{1,p}(\mathbf{R}_+,\omega)$, new results from complex interpolation theory dealing with interpolation spaces between weighted Dirichlet spaces, and for deriving new Morrey type embedding theorems for our Dirichlet space. Similar results were obtained for weigted Sobolev space

$$W^{1,p}(\mathbf{R}_+,t^\beta) = \{u \in L^p(\mathbf{R}_+,t^\beta) : u^{'} \in L^p(\mathbf{R}_+,t^\beta)\}, \text{ where } \beta \in \mathbf{R}.$$

The talk will be based on results obtained together with Claudia Capone and Radosław Kaczmarek.

References

[1] C. Capone, A. Kałamajska, *Asymptotics, Trace, and Density Results for Weighted Dirichlet Spaces Defined on the Half-line*, Potential Analysis, 60 (2024), no.4, 1301–1331, https://doi.org/10.1007/s11118-023-10089-2

[2]R. Kaczmarek, A. Kałamajska, *Density results and trace operator in weighted Sobolev spaces defined on the half-line, equipped with power weights*, Journal of Approximation Theory 291 (2023). Art. No. 105896, https://doi.org/10.1016/j.jat.2023.1058

List of Abstracts – Invited Speakers

Overview of Musielak-Orlicz-Sobolev spaces

Anna Kamińska **IS**

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It is an overview of Musielak-Orlicz-Sobolev spaces (MOS spaces) that are Sobolev spaces defined within the frame of Musielak-Orlicz spaces. In particular they become Orlicz-Sobolev spaces or variable exponent Sobolev spaces dependently on the choice of the Musielak-Orlicz function. Recently MOS spaces have gained some interest in studies of their geometric structure as well as in applications to PDE. We will present the results on density of the space of compactly supported functions in MOS and if time allows some geometric properties of those spaces.

List of Abstracts – Invited Speakers

Henryk Hudzik (1945–2019). With memories and appreciation.

Lech Maligranda IS

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Henryk Hudzik was my friend and we wrote 8 joint papers in the years 1990–2004. So I feel obliged to tell about him and a little about his mathematics. First, I will present the history of Hudzik's life. I will also recall his scientific career with a doctorate in 1977, a habilitation in 1986 and a professorship from 1992. He worked at the University of Adam Mickiewicz in Poznań in the years 1972–2015. Hudzik's research interests include functional analysis, in particular: the theory of function spaces, Banach lattices, the geometry of Banach spaces and its applications, operator theory, metric fixed point theory. The number of his publications includes over 220 research papers, 3 chapters in monographs and several popular science and review papers. I will mention three of his most important papers, in my opinion.

In the years 1986–2020 I worked outside Poland, but we met many times in Poznań and other countries, which I will show in many joint photos.

References

[1] L. Maligranda and W. Wnuk, *100 lat matematyki na Uniwersytecie w Poznaniu 1919–2019* [*100 years of mathematics at the University in Poznań 1919–2019*], Wydawnictwo Naukowe UAM, Poznań 2021, 658 pages with 368 photos.

[2] M. Mastyło and J. Musielak, *Henryk Hudzik – vita et opera*, Comment. Math. 55 (2015), no. 2, 45–78.

List of Abstracts – Invited Speakers

Convergence results for varying measures

Valeria Marraffa

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Some limit theorems of the type

$$\int_\Omega f_n \, dm_n \to \int_\Omega f \, dm$$

are presented for scalar, (vector), (multi)-valued sequences of m_n-integrable functions f_n. Conditions for the convergence of sequences of measures $(m_n)_n$ and of their integrals $(\int f_n dm_n)_n$ in a measurable space Ω are of interest in many areas of pure and applied mathematics such as statistics, transportation problems, interactive partial systems, neural networks and signal processing.
Sufficient conditions in order to obtain some kind of Vitali's convergence theorems for a sequence of (multi)functions $(f_n)_n$ integrable with respect to a sequence of measures $(m_n)_n$ are considered.
We consider the asymptotic properties of $(\int_\Omega f_n dm_n)_n$ with respect to varying measures, which are setwisely or vaguely convergent in an arbitrary measurable spaces.
Consequently, a continuous dependence result for a wide class of differential equations with many interesting applications, namely measure differential equations (including Stieltjes differential equations, generalized differential problems, impulsive differential equations with finitely or countably many impulses and also dynamic equations on time scales) is provided.

References

[1] L. Di Piazza, V. Marraffa, K. Musiał, A. Sambucini, *Convergence for varying measures*, J. Math. Anal. Appl., Vol. 518, N.2, Paper N. 126782, (2023).

[2] L. Di Piazza, V. Marraffa, K. Musiał, A. Sambucini, *Convergence for varying measures in the topological case*, Annali di Matematica Pura e Applicata, (4) 203 (2024) 71-86.

[3] V. Marraffa, B. Satco, *Convergence Theorems for Varying Measures Under Convexity Conditions and Applications*, Mediterr. J. Math., (2022), 19:274.

Multilinear fractional integrals: boundedness criteria and sharp estimates

Alexander Meskhi IS

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Necessary and sufficient conditions for the trace inequality of multilinear fractional integral operators from a product of Lebesgue/ Morrey spaces to a Lebesgue/ Morrey space will be presented. Sharp Olsen-type estimates in the multilinear setting will also be discussed. Similar problems for one-sided fractional integral operator are studied as well.

The talk is based on the papers [1-5]

References

[1] L. Grafakos and A. Meskhi, *On sharp Olsen's and trace inequalities for multilinear fractional integrals*, Potential Analysis, 59(2023), 1039-1050, https://doi.org/10.1007/s11118-022-09991-y.

[2] G. Imerlishvili, A. Meskhi and Q. Xue, *Multilinear Fefferman-Stein type inequality and its generalizations*, Trans. A. Razmadze Math. Inst. 174 (2020), no. 1, 83-92.

[3] V. Kokilashvili, M. Mastylo and A.Meskhi, *On the Boundedness of Multilinear Fractional Integral Operators*, J. Geometric Anal. 30 (2020), 667-679, https://doi.org/10.1007/s12220-019-00159-6.

[4] V. Kokilashvili and A. Meskhi, *A Complete characterization of the generalized multilinear Sobolev inequality in grand product Lebesgue spaces defined on non-homogeneous spaces*, Results in Math. 78, 181 (2023), DOI : 10.1007/s00025-023-01959-7.

[5] A. Meskhi, *Boundedness weighted criteria for multilinear Riemann-Liouville integral operators*, Trans. A. Razmadze Math. Inst. 177 (2023), No. 1, 147–148.

List of Abstracts – Invited Speakers

Polynomial projections onto the lines in L^p and simultaneous minimal extensions

Grzegorz Lewicki, Michael Prophet IS

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With $1 \leq p \leq \infty$, let $L^p := L^p[-1,1]$ denote, as usual, the space of (extended) real-valued functions $f : [-1,1] \to \overline{\mathbb{R}}$ for which $|f|^p$ is Lebesgue integrable; let $V \subset X$ denote that space of lines. It is only for the cases $p = 1, 2, \infty$ where a projection of minimal operator norm (a *minimal projection*) from X onto V is known. For other values of p, our investigations using polynomial subspace of L^p has yielded unexpected and interesting results for minimal operator extensions. Indeed we find a context for the determination of *simultaneous minimal extensions* of operators. We provide several applications using polynomial subspaces of L^4 projected onto V. Moreover, this work indicates, for some values of p, a reasonable approach to determining minimal projections from L^p on V.

Non-linear variants of factorization theorems for operators on Banach function spaces

Enrique A. Sánchez Pérez IS

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This talk explores several approaches aimed at extending classical factorization results for linear operators to nonlinear ones. These classical results are building blocks of the current theory of (linear) operators on Banach function spaces. We will present some results on two relevant classes of nonlinear maps: multilinear maps and Lipschitz maps. From a technical point of view, our results could be understood in terms of what kind of separation argument is used to obtain factorization theorems: summability inequalities for operators (involving Pietsch-type theorems and Pisier-type theorems), Maurey-Rosenthal factorizations through various classical function spaces characterized by geometric lattice inequalities, and Nikishin-type results involving some notion of type for domain spaces. The most fruitful class of function spaces we have used are Orlicz spaces and some variants and generalizations of these spaces, such as Marcinkiewicz spaces. Our goal is to provide an overview of the state of the art of factorization techniques concerning nonlinear operators on Banach function spaces.

Very rough function spaces and global solutions for nonlinear evolution equations

Baoxiang Wang IS

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We introduce a class of very rough function spaces including $E^{\sigma,s}$ for which the norms are defined by

$$\|f\|_{E^{\sigma,s}} = \|2^{s|\xi|}\langle\xi\rangle^{\sigma}\,\widehat{f}(\xi)\|_{L^2},\ s < 0.$$

We obtain the existence and uniqueness of the global solutions for the Navier-Stokes, Euler, nonlocal NLS, NLKG equations if the initial data belong to $E^{\sigma,s}$ and their Fourier transforms are supported in the cone-like domains. We have no smallness condition on initial data. This talk is a survey of our recent joint works with J. Chen, H. Feichtinger, K. Gröchenig, K. Li, Y. Lu, K. Nakanishi, Z. Wang.

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Professor Julian Musielak, an outstanding member of the Poznań School of Mathematics

Witold Wnuk IS

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Professor Julian Musielak was a person whose achievements to develop of mathematical investigations and the Poznań School of Mathematics were important, essential, valuable, inspired, numerous and varied. His scientific interests concerned many problems related to functional analysis, approximations and expansions, harmonic analysis on Euclidean spaces, sequences, series, summability, measure and integration, real functions, history and biography. At the end of fifties of twentieth century Julian Musielak, together with Władysław Orlicz, initiated investigations of spaces of measurable functions called later one Musielak-Orlicz spaces $L^{\psi}(S, \Sigma, \mu)$ (they generalized Nakano's ideas considering non-convex functions ψ). During his sixty years of activity, he published over two hundred papers including monographs and textbooks (looking at Mathematical Reviews you can see, that they have been cited 1.695 times – data from the end of May 2024). Having so broad knowledge Professor Musielak was able to take care about many pupils preparing their Ph.D. theses devoted to various topics (he was a supervisor of thirty six dissertations). Professor Musielak had a lot of international relations and he was a desirable collaborator in research made by other mathematicians (twenty five is a number of his co-authors). He was also a fan of traveling. Towards the end of his life he described trips in extensive memories, where he also told the events of his life. The main aim of the lecture is to show how unique Professor Musielak was.

Numerical calculations of p-Amemiya norm in Orlicz function spaces for $1 \leq p \leq \infty$

Adam Bohonos CT

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A map $\Phi : \mathbb{R} \to [0, \infty]$ is said to be an *Orlicz function* if it is even, convex, left continuous on the whole $\mathbb{R}_+$, $\Phi(0) = 0$ and Φ is not identically equal to zero. It is worth to mention that $\Phi(u) = \infty$ for $u < \infty$ is not excluded.
For any $x \in L^0(T)$ define the $p - Amemiya$ norm ($p \in [1, \infty]$) by the formula

$$\|x\|_{\Phi,p} = \inf_{k>0} \frac{1}{k} s_{\Phi,p}(kx),$$

where

$$s_{\Phi,p}(u) = \begin{cases} (1 + (I_\Phi(u))^p)^{\frac{1}{p}} & \text{for} \quad 1 \leq p < \infty \\ \max\{1, I_\Phi(u)\} & \text{for} \quad p = \infty \end{cases}$$

and

$$I_\Phi(u) = \int_T \Phi\left(u(t)\right) d\mu(t).$$

It is known that $\|x\|_{\Phi,1}$ coincide with Orlicz and $\|x\|_{\Phi,\infty}$ coincide with Luxemburg norm.

Using Python with libraries NumPy and Matplotlib we can, among other things, automate calculations $\|x\|_{\Phi,p}$ and plot the graph of $\frac{1}{k} s_{\Phi,p}(kx)$ for any Φ, p, and x (for $\mu(T) < \infty$) (Figure 1).

References

[1] Y. Cui, L. Duan, H. Hudzik, M. Wisła, *Basic theory of p-Amemiya norm in Orlicz spaces* $(1 \leq p \leq 1)$: *Extreme points and rotundity in Orlicz spaces equipped with these norms*, Nonlinear Anal. 69 (2008), 1796-1816.

[2] C. R. Harris et al. , *Array programming with NumPy*, Nature 585 (7825) (2020), 357–362.

[3] J. D. Hunter, *Matplotlib: A 2D graphics environment*, Computing in Science & Engineering 9 (3) (2007), 90–95.

[4] M. Wisła, *Orlicz spaces equipped with s-norms*, J. Math. Anal. Appl. 483 (2).

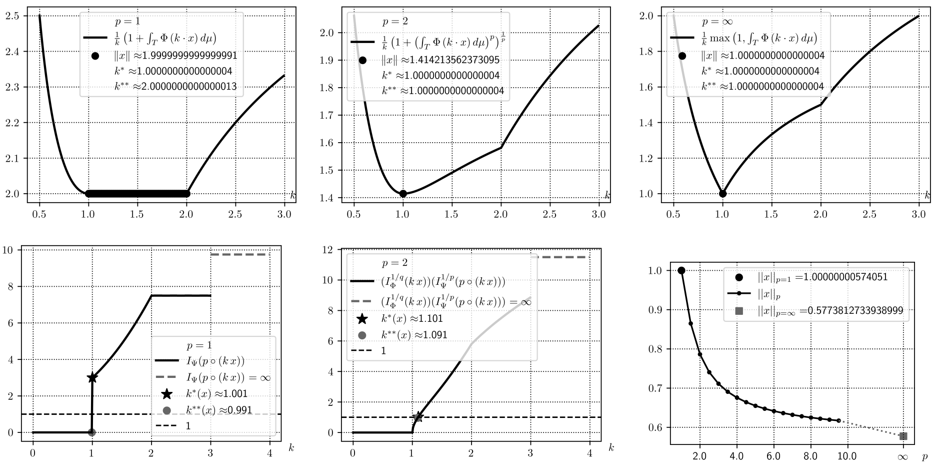

Figure 0.1: Some plots obtained with Python, and own module

About function spaces related to fractional order operators

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In this lecture, we will present the problem of finding invariant spaces for a wide class of fractional order integral operators. This research began with the results of [1] for the Riemann-Liouville operator, and continued in subspaces of continuous functions [2], [3]. We will present results on the study of such spaces as Banach algebras [4], as well as constructions of such invariant spaces for a large class of generalized integral operators.

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List of Abstracts – Talks

Substochastic operators in symmetric spaces

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By L^0 we denote a set of all (equivalence classes of) extended real valued m-measurable functions on $I = [0, \alpha)$, where $0 < \alpha \leq \infty$. Assuming that $x \in L^0$ we define $x^*(t) = \inf\{\lambda > 0 : m(|x| > \lambda) \leq t\}$, $x^{**}(t) = \frac{1}{t}\int_0^t x^*(s)ds$ for $t > 0$. For any x, y in $L^1 + L^\infty$, the *Hardy-Littlewood-Pólya relation* $\prec$ is given by

$$x \prec y \Leftrightarrow x^{**}(t) \leq y^{**}(t) \text{ for all } t > 0.$$

A symmetric space E is said to be *uniformly K-monotone* (shortly $E \in (UKM)$) if for any $(x_n), (y_n) \subset E$ such that $x_n \prec y_n$ and $\lim_{n\to\infty} \|x_n\|_E = \lim_{n\to\infty} \|y_n\|_E < \infty$ we have $\|x_n^* - y_n^*\|_E \to 0$ as $n \to \infty$. A symmetric space E is called *decreasing uniformly K-monotone*, shortly $E \in (DUKM)$ (resp. *increasing uniformly K-monotone*, shortly $E \in (IUKM)$) if for any $(x_n), (y_n) \subset E$ such that $x_{n+1} \prec x_n \prec y_n$ (resp. $x_n \prec y_n \prec y_{n+1}$) for all $n \in \mathbb{N}$ and $\lim_{n\to\infty} \|x_n\|_E = \lim_{n\to\infty} \|y_n\|_E < \infty$ we have $\lim_{n\to\infty} \|x_n^* - y_n^*\|_E = 0$.

An operator T from a Banach function space $(X, \|\cdot\|_X)$ into a Banach function space $(Y, \|\cdot\|_Y)$ is said to be *positive contraction* if its norm is at most one and it satisfies the property $T(x) \geq 0$ whenever $x \geq 0$. An admissible operator for a Banach couple (L^1, L^∞) is called a *substochastic operator* if it is positive contraction on L^1 and L^∞ (equivalently T is *substochastic* if and only if $T(x) \prec x$ for all $x \in L^1 + L^\infty$ (see [1]).

We discuss complete criteria for which increasing uniform K-monotonicity and lower locally uniform K-monotonicity are equivalent in symmetric spaces. Next, we investigate a connection between K-monotonicity properties and the convergence of a sequence of substochastic operators in a norm of the symmetric space. Finally, we research compactness of admissible operators under additional assumptions in symmetric spaces. For more details please see [2].

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Lipschitz continuity and continuity for bounded operators

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In this talk, we explore Lipschitz and α-Hölder type continuity inequalities for means of positive bounded operators on a Hilbert space H, considering the norm topology induced by unitarily invariant norms. Specifically, for $A, B, C \in B(H)$ we examine inequalities of the form

$$\|A\sigma C - B\sigma C\| \leq \lambda(C)\|f(|A-B|)\|,$$

where $\lambda(C)$ is a constant depending on C and $\|\cdot\|$ is a unitatrily invariant norm on $B(H)$.

Our focus includes the weighted geometric means,

$$A\sharp_\alpha B = A^{1/2}(A^{-1/2}BA^{-1/2})^\alpha A^{1/2}, \quad \alpha \in [0,1],$$

general Kubo-Ando means [1] with representing operator monotone function function f_σ

$$A\sigma B = A^{1/2} f_\sigma(A^{-1/2}BA^{-1/2})A^{1/2},$$

as well as Rényi means [2]

$$Q_{\alpha,z}(A,B) = (B^{\frac{1-\alpha}{2z}} A^{\frac{\alpha}{z}} B^{\frac{1-\alpha}{2z}})^z.$$

It is a joint work with Raluca Dumitru and Jose Franco from the University of North Florida.

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List of Abstracts – Talks

On maximal projection constants

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Let X be a Banach space over $\mathbb{K}$, where $\mathbb{K} = \mathbb{R}$ or $\mathbb{K} = \mathbb{C}$. Let $Y \subset X$ be a subspace. By $\mathcal{P}(X, Y)$ denote the set of all linear and continuous projections from X onto Y, recalling that an operator $P \colon X \to Y$ is called a *projection* onto Y if $P|_Y = \mathrm{Id}_Y$. We define the *relative projection constant* of subspace Y of space X by

$$\lambda(Y, X) := \inf\{\|P\| : \ P \in \mathcal{P}(X, Y)\}.$$

Now we can define the *absolute projection constant* of Y by

$$\lambda(Y) := \sup\{\lambda(Y, X) : Y \subset X\}.$$

The ultimate goal of researchers in this area is to determine the exact value of *maximal absolute projection constant*, which is defined by

$$\lambda_{\mathbb{K}}(m) := \sup\{\lambda(Y) : \ \dim(Y) = m\}.$$

In 1994, H. König and N. Tomaczak-Jaegermann stated the following estimation.

Theorem (stated in [3]; proved in [1]) *Let $m > 1$ then*

i) $\lambda_{\mathbb{R}}(m) \leq \frac{2}{m+1}\left(1 + \frac{m-1}{2}\sqrt{m+2}\right)$

ii) $\lambda_{\mathbb{C}}(m) \leq \frac{1}{m}\left(1 + (m-1)\sqrt{m+1}\right)$.

Unfortunately, their proof is based on an erroneous lemma, as was pointed out in [2]. In this talk, we will present the correct proof of the latter. Moreover relying on this result we provide the exact values of $\lambda_{\mathbb{K}}(m)$ in cases where the maximal equiangular tight frame exists in $\mathbb{K}^m$.

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s-numbers and entropy numbers of general diagonal operators

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s-numbers and entropy numbers of Sobolev embeddings find applications in PDE's. However estimating them for function spaces embeddings is not easy but usually we can finally reduce this problem to study asymptotic behaviour of s-numbers and entropy numbers of diagonal operators in p-spaces. First we using the technique of discretization by wavelet bases, atomic or subatomic decompositions. Then we can reduce the problem to the corresponding problem for respective sequence spaces. However the resulting sequence spaces, are still quite complicated, often they are of mixed-norm type and/or involve weights. Therefore a further reduction is necessary, which by factorization leads to diagonal operators in p-spaces. Therefore, the study of s-numbers and entropy numbers of general diagonal operators is very useful.

List of Abstracts – Talks

Delta-convexity with arbitrarily given weights

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Generalizing Theorem 1.1 from author's paper [3] D. S. Marinescu and M. Monea have proved, among others, the following result (see [4, Theorem 2.7]).

Theorem M-M. *Let $\varphi : (a,b) \longrightarrow \mathbb{R}$ be a differentiable function and let $f : (a,b) \longrightarrow \mathbb{R}$ be a convex function admitting some weights $s,t \in (0,1)$ such that the inequality*

$$(*) \qquad tf(x) + (1-t)f(y) - f(sx + (1-s)y) \le t\varphi(x) + (1-t)\varphi(y) - \varphi(sx + (1-s)y)$$

is satisfied for all $x, y \in (a,b)$. Then the function f is differentiable and the inequality

$$|f'(x) - f'(y)| \le |\varphi'(x) - \varphi'(y)|$$

holds true for all $x, y \in (a,b)$.

Note that in the case where $s = t = \frac{1}{2}$, the convexity assumption imposed upon f in the above result renders $(*)$ to be equivalent to

$$\left| \frac{f(x)+f(y)}{2} - f\left(\frac{x+y}{2}\right) \right| \le \frac{\varphi(x)+\varphi(y)}{2} - \varphi\left(\frac{x+y}{2}\right), \quad x, y \in (a,b),$$

defining (in the class of continuous functions) the notion of delta convexity in the sense of L. Veselý and L. Zajíček (see [6]).

Without any convexity assumption we offer the following counterpart of Theorem M-M for vector valued mappings.

Theorem 2. *Given an open interval $(a,b) \subset \mathbb{R}$, a normed linear space $(E, \|\cdot\|$, and two real numbers $s,t \in (0,1)$ (weights) assume that a map $F : (a,b) \longrightarrow E$ is delta (s,t)-convex with a differentiable control function $f : (a,b) \longrightarrow \mathbb{R}$, i.e. that a functional inequality*

$$\|tF(x) + (1-t)F(y) - F(sx + (1-s)y)\| \le tf(x) + (1-t)f(y) - f(sx + (1-s)y)$$

is satisfied for all $x, y \in (a,b)$. If the function

$$(a,b) \ni x \longmapsto \|F(x)\| \in \mathbb{R}$$

is upper bounded on a set of positive Lebesgue measure, then F is differentiable and the inequality

$$\|F'(x) - F'(y)\| \le |f'(x) - f'(y)|$$

holds true for all $x, y \in (a,b)$.

Among other consequences of Theorem 2 one may encounter the following result.

Theorem 3. *Given an open interval* $(a, b) \subset \mathbb{R}$, *a normed linear space* $(E, \|\cdot\|)$ *that is reflexive or constitutes a separable dual space, and two weights* $s, t \in (0, 1)$, *assume that a map* $F : (a, b) \longrightarrow E$ *is delta* (s, t)*-convex with a* C^2*-control function* $f : (a, b) \longrightarrow \mathbb{R}$. *If the function*

$$(a, b) \ni x \longmapsto \|F(x)\| \in \mathbb{R}$$

is upper bounded on a set of positive Lebesgue measure, then F *is twice differentiable almost everywhere in* (a, b) *and the domination*

$$\|F''(x)\| \leq f''(x)$$

holds true for almost all $x \in (a, b)$.

The assumption that a normed linear space $(E, \|\cdot\|)$ spoken of in Theorem 3 is reflexive or constitutes a separable dual space may be replaced by a more general requirement that $(E, \|\cdot\|)$ has the Radon-Nikodym property (RNP), i.e. that every Lipschitz function from $\mathbb{R}$ into E is differentiable almost everywhere (see Y. Benyamini and J. Lindenstrauss [1]). This definition (of Rademacher type character) is not commonly used but is more relevant to the subject of the present paper. R.S. Phillips [5] showed that reflexive Banach spaces enjoy the RNP whereas N. Dunford and B.J. Pettis [2] proved that separable dual spaces have the RNP.

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Maximal Demyanov difference

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Demyanov difference [1,2,3] of compact convex sets is maximal if it coincides with Minkowski difference. It is a known result [4,5] that in the plane maximal Demyanov difference of sets implies maximal number of vertexes of Minkowski sum of sets. We extend this result to the following cases of two polytopes 1) one polytope is two-dimensional i.e. it is a polygon and 2) both polytopes are contained in a three-dimensional space and one of them is a tetrahedron. We also give an example of two tetrahedra i.e. three-dimensional simplices in four-dimensional space which have maximal Demyanov difference but do not have Minkowski sum with maximal number of vertexes.

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On the interpolation constants for variable Lebesgue spaces

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In the late 1970's, Musielak, Hudzik and Urbański [1] proved the interpolation theorem of Riesz-Thorin type for variable Orlicz spaces (usually called Musielak-Orlicz spaces) with the interpolation constant 4. In this talk we consider the question of optimality of the interpolation constant in the case of variable Lebesgue spaces. For $\theta \in (0,1)$ and variable exponents $p_0(\cdot), q_0(\cdot)$ and $p_1(\cdot), q_1(\cdot)$ with values in $[1,\infty]$, let the variable exponents $p_\theta(\cdot), q_\theta(\cdot)$ be defined by

$$1/p_\theta(\cdot) := (1-\theta)/p_0(\cdot) + \theta/p_1(\cdot), \quad 1/q_\theta(\cdot) := (1-\theta)/q_0(\cdot) + \theta/q_1(\cdot).$$

The Riesz-Thorin type interpolation theorem for variable Lebesgue spaces says that if a linear operator T acts boundedly from the variable Lebesgue space $L^{p_j(\cdot)}$ to the variable Lebesgue space $L^{q_j(\cdot)}$ for $j = 0, 1$, then

$$\|T\|_{L^{p_\theta(\cdot)} \to L^{q_\theta(\cdot)}} \le C \|T\|^{1-\theta}_{L^{p_0(\cdot)} \to L^{q_0(\cdot)}} \|T\|^{\theta}_{L^{p_1(\cdot)} \to L^{q_1(\cdot)}},$$

where C is an interpolation constant independent of T. We consider two different modulars $\varrho^{\max}(\cdot)$ and $\varrho^{sum}(\cdot)$ generating variable Lebesgue spaces and give upper estimates for the corresponding interpolation constants $C_{\max}$ and $C_{\rm sum}$, which imply that $C_{\max} \le 2$ and $C_{sum} \le 4$, as well as, lead to sufficient conditions for $C_{\max} = 1$ and $C_{sum} = 1$. We also construct an example showing that, in many cases, our upper estimates are sharp and the interpolation constant is greater than one, even if one requires that $p_j(\cdot) = q_j(\cdot)$, $j = 0, 1$ are Lipschitz continuous and bounded away from one and infinity (in this case $\varrho^{max}(\cdot) = \varrho^{sum}(\cdot)$). The talk is based on the joint work wtih Eugene Shargorodksy [2].

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The spaces Ces_∞ and ces_∞ are not isometric, yet still isomorphic

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Recall that the **Cesáro function space** Ces_∞ is defined as the collection of all measurable functions f such that the norm $\|f\|_{Ces_\infty} := \sup_{x>0} \frac{1}{x} \int_0^x |f(t)|\, dt$ is finite. The sequence counterpart of this construction, called the **Cesáro sequence space** ces_∞, is understood as the collection of all sequences $x = \{x_n\}_{n=1}^\infty$ such that the norm $\|x\|_{ces_\infty} := \sup_{n\in\mathbb{N}} \frac{1}{n} \sum_{k=1}^n |x_k|$ is finite.

From the perspective of the remarkable result due to Aleksander Pełczyński [4], later complemented by Denny Leung [3], it is completely natural to ask whether the spaces Ces_∞ and ces_∞ are isomorphic. In fact, this question was asked by Astashkin and Maligranda in [2] (precisely, see Problem 1 ibidem) and finally answered positively by Astashkin, Leśnik and Maligranda in (Theorem 5.1)[1]. The goal of this talk is to provide a completely different proof of this fact. We will also show that although isomorphic, the spaces Ces_∞ and ces_∞ are not isometric.

This talk is based on a joint work with Jakub Tomaszewski titled *"Rethinking the isomorphism between Ces_∞ and ces_∞"* (at the time of writing this text, it is still a work is in progress).

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Amenability of Banach algebra-valued ℓ_p-sequence algebras

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The definition of an amenable Banach algebra was introduced after Johnson's theorem (1972) which provides an equivalent condition for amenability of locally compact groups in terms of convolution Banach algebras and derivations. In this talk we will focus on amenability properties of vector-valued sequence algebras. Namely, we will investigate the relation between amenability properties of a Banach algebra A and those of $\ell_p(A)$, where $1 \leq p < \infty$.

Recall that in [7] it is shown that for a compact Hausdorff space K and a Banach algebra A, Banach algebra $C(K, A)$ (A-valued continuous functions on X) is (weakly) amenable if and only if so is A (weak amenability proved only for commutative A). In case when A is a C^*-algebra (thus weakly amenable by [2, Corollary 4.2]), weak amenability of $\ell_1(A)$ was implicitly proved in [5] and [6]. Also characterization of amenable $\ell_\infty(A)$ was given in [4, Theorem 2.5] for C^*-algebras A. It is also known that if A is a Banach algebra then $c_0(A)$ is amenable if and only if A is amenable.
The talk is based on a joint work [3] and is a part of an ongoing project.

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List of Abstracts – Talks

Geometric properties of noncommutative Orlicz spaces with p-Amemiya norms in the context of metric and Brègman projections

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Combining known characterisations of several geometric properties of Orlicz spaces with p-Amemiya norms with the results translating between the geometric properties of commutative and noncommutative rearrangement invariant spaces (cf. [4] for a review), we obtain the corresponding geometric characteristics of noncommutative Orlicz spaces with p-Amemiya norms. Next, we use them for:

1) the characterisation of strictly convex and reflexive (commutative and noncommutative) Orlicz spaces with p-Amemiya norms, as well as commutative and noncommutative Orlicz–Lorentz spaces with Amemiya and Morse–Transue–Nakano norms, by the norm-to-norm continuity of metric projections (substantially generalising the result of Wáng and Chén [8];

2) providing sufficient conditions for the (norm-to-norm, uniform, and Lipschitz–Hölder) continuity of (left and right) Brègman projections, as well as for the existence of monoids of (left and right) Brègman strongly quasi-nonexpansive operators (including Brègman resolvents of monotone operators), for a large family of the Vaĭnberg–Brègman relative entropies over reflexive Banach spaces determined by the integrals Ψ_φ of strictly increasing ϕ-functions φ, with concrete examples specified using the above results on the Orlicz space geometry;

3) establishing sufficient conditions for q-uniform convexity and r-uniform Fréchet differentiability of a family of nonassociative Orlicz spaces (over type II$_1$ JBW-algebras) with the Morse–Transue–Nakano norm, determined by the family of Orlicz functions given by positive univariate monomials; from this we derive the continuity properties of metric and Brègman projections over these spaces (these are the first new results on nonassociative Orlicz spaces since their introduction by Tadzhibaev [6]).

Ad rem 2): Under generalisation from Hilbert spaces to Banach spaces $(X, \|\cdot\|_X)$, metric projections lose several properties that are crucial for their applications (such as: pythagorean and cosine theorems, monotonicity and nonexpansivity, and strong convergence of iterated projections). However, under replacement of the metric distance by the Vaĭnberg–Brègman relative entropy, $D_\Psi(x, y) := \Psi(x) - \Psi(y) - (\Psi'(y))(x - y)$ [7,3] (where Ψ is a proper, convex, lower semicontinuous function on X, with Gateaux derivative Ψ'), these properties hold for reflexive Banach spaces (for suitable conditions on Ψ), while coinciding with metric setting in Hilbert spaces (cf.: [1] for the main ideas with $\Psi = \frac{1}{2}\|\cdot\|_X^2$; [5] for a review with any Ψ). Yet, so far, the availability of concrete functions Ψ, satisfying all relevant conditions, has been very limited.

We use Orlicz space theory for the analogies underlying the study of $\Psi = \Psi_\varphi$, and as a source of examples of $(X, \|\cdot\|_X)$ differentiating between various properties of Ψ_φ. (While the geometric properties of Orlicz spaces are characterised by the properties of the Young functions $\Phi(u) = \int_0^u \mathrm{d}t\, f(t)$, the geometric properties of $(X, \|\cdot\|_X)$ are characterised [2,9] by the properties of $\Psi_\varphi(x) := \int_0^{\|x\|_X} \mathrm{d}t\, \varphi(t)$, where $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is strictly increasing, continuous, $\varphi(0) = 0$, and $\lim_{t\to\infty} \varphi(t) = \infty$.)

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On attaining diameter two and some related properties in Banach spaces

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We study versions of the diameter two properties where the diameter of slices, weakly relatively open subsets, or convex combinations of slices of the unit ball is attained. We also introduce some new related properties in Banach spaces and show connections between them. We characterize one of these properties in the class of Musielak–Orlicz spaces over a complete σ-finite measure space.

Gagliardo-Nirenberg inequality via a new pointwise estimate

Karol Leśnik CT

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I will present a new type of pointwise estimate of the Kałamajska–Mazya–Shaposhnikova type, where averaging operators replace the maximal operator. It allows us to extend the classical Gagliardo–Nirenberg interpolation inequality to all rearrangement invariant Banach function spaces without any assumptions on their Boyd indices. The applied method is new in this context and may be seen as a kind of sparse domination technique fitted to the context of r.i. Banach function spaces. The talk is based on a joint work with Tomas Roskovec and Filip Soudsky.

List of Abstracts – Talks

On the value of the fifth maximal projection constant

Barbara Lewandowska CT

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Let X be a Banach space over $\mathbb{K}$, where $\mathbb{K} = \mathbb{R}$ or $\mathbb{K} = \mathbb{C}$. Let $Y \subset X$ be a subspace. By $\mathcal{P}(X, Y)$ denote the set of all linear and continuous projections from X onto Y, recalling that an operator $P\colon X \to Y$ is called a *projection* onto Y if $P|_Y = \mathrm{Id}_Y$. We define the *relative projection constant* of a subspace Y of a space X by

$$\lambda(Y, X) := \inf\{\|P\| : \ P \in \mathcal{P}(X, Y)\}.$$

Now we can define the *absolute projection constant* of Y by

$$\lambda(Y) := \sup\{\lambda(Y, X) : Y \subset X\}.$$

The ultimate goal of researchers in this area is to determine the exact value of *maximal absolute projection constant*, which is defined by

$$\lambda_{\mathbb{K}}(m) := \sup\{\lambda(Y) : \ \dim(Y) = m\}.$$

In 1960, B.Grünbaum conjectured that $\lambda_{\mathbb{R}}(2) = \frac{4}{3}$ (see [6]), and only in 2010, B. Chalmers and G. Lewicki proved it (see [2]) and that was the only known nontrivial case. Recently, we have provided exact values of $\lambda_{\mathbb{K}}(m)$ in cases where the maximal equiangular tight frame exists in $\mathbb{K}^m$. There are numerous examples of complex maximal ETFs, for example, for $m \in \{1, \dots, 17, 19, 24, 28, 35, 48\}$ (see, e.g., [5]). In fact, it is conjectured that there is a complex maximal ETF in every dimension (Zaurner's conjecture [7]). Unlike in the complex case, real maximal ETFs seem to be rare objects. The only known cases are for m equal to $2, 3, 7$ and 23. A lot of the community believes that these are all real cases where maximal ETFs exist. In other cases, the determination of the constant $\lambda_{\mathbb{R}}(m)$ seems to be difficult. Numerical experiments conducted by B. L. Chalmers (and unfortunately unpublished) suggest that $\lambda(5) \approx 2.06919$. In this talk, relying on a new construction of certain mutually unbiased equiangular tight frames, we will provide the lower bound for $\lambda_{\mathbb{R}}(5)$ and present some arguments that it may be its true value.

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Embeddings of generalized Besov-type and Tribel–Lizorkin-type spaces

Zhen Liu

CT

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Let $0 < p < \infty$, $0 < q \leq \infty$, and $s \in \mathbb{R}$. We introduce a new type of generalized Besov-type spaces $B^{s,\varphi}_{p,q}(\mathbb{R}^d)$ and generalized Triebel–Lizorkin-type spaces $F^{s,\varphi}_{p,q}(\mathbb{R}^d)$, where φ belongs to the class $\mathcal{G}_p$, that is, $\varphi : (0,\,\infty) \to (0,\,\infty)$ is non-decreasing and $t^{-d/p}\varphi(t)$ is non-increasing in $t > 0$.

In this talk, we study embeddings between generalized Triebel–Lizorkin-type spaces $F^{s,\varphi}_{p,q}(\mathbb{R}^d)$ and generalized Besov-type spaces $B^{s,\varphi}_{p,q}(\mathbb{R}^d)$. Our approach requires a wavelet characterisation of those spaces which we establish for the system of Daubechies' wavelets. Then we prove necessary and sufficient conditions for the embedding $F^{s_1,\varphi_1}_{p_1,q_1}(\mathbb{R}^d) \hookrightarrow F^{s_2,\varphi_2}_{p_2,q_2}(\mathbb{R}^d)$. We also provide an almost final answer to the embeddings within the scales $B^{s,\varphi}_{p,q}(\mathbb{R}^d)$.

This is joint work with Dorothee D. Haroske (Jena), Susana D. Moura (Coimbra) and Leszek Skrzypczak (Poznań).

Seminormwise approximation by matrix means of Fourier series

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Let $X = L^p$ or $X = C$, where L^p $(1 \leq p \leq \infty)$ $[C]$ be the class of all 2π–periodic real–valued functions, integrable in the Lebesgue sense with p–th power when $p \geq 1$ and essentially bounded when $p = \infty$ [continuous] over $Q = [-\pi, \pi]$ with the usual norm.

For a seminorm P, we define the following seminormed space:

$$(X, P) = \{f \in X : P(f) < \infty\},$$

with the property that $f(\cdot + h) \in (X, P)$ for any $h \in \mathbb{R}$ and the following conditions:

1. for any $h \in \mathbb{R}$

$$P(f(\cdot + h)) = P(f(\cdot)),$$

2. if $|f(x)| \leq |g(x)|$ for every $x \in [-\pi, \pi]$, then

$$P(f) \leq P(g).$$

Consider the trigonometric Fourier series Sf with the partial sums$S_k f$ and its tranformation

$$T_{n,A} f(x) := \sum_{k=0}^{\infty} a_{n,k} S_k f(x)$$

by a matrix $A := (a_{n,k})_{0 \leq n,k < \infty} = (a_{n,k})$ of regular summability method.

The main aim of this paper is to estimate the seminormwise deviation:$P\left(T_{n,A} f - f\right)$ depending on the following r, s-th differences (higher forward differences):

$$A_{\mu,\nu}^{r,s}(b_{\cdot}) = A_{\mu,\nu}^{r,s}\left(\frac{a_{n,\cdot}}{b_{\cdot}}\right) = \sum_{k=\mu}^{\nu} b_k \left|\Delta_r^s\left(\frac{a_{n,k}}{b_k}\right)\right|,$$

where $\Delta_r^0(a_{n,k}) = a_{n,k},\ \Delta_r^1(a_{n,k}) = a_{n,k} - a_{n,k+r}$ and $\Delta_r^s(a_{n,k}) = \Delta_r^1\left(\Delta_r^{s-1}(a_{n,k})\right)$,

for $r, s \in \mathbb{N}$ and $\mu, \nu, k, n \in \mathbb{N}_0$, with positive nondecreasing sequence (b_k) .

As a measure of the such deviation we will use the classical modulus of continuity of f in the spaces (X, P) defined by the formula

$$\omega f\left(\delta\right)_P = \sup_{0\le t\le \delta} P\left(\varphi_{\cdot}\left(t\right)\right),$$

where

$$\varphi_x\left(t\right) = f\left(x+t\right) + f\left(x-t\right) - 2f\left(x\right).$$

P. Chandra in [1] and [2] estimated the deviation $T_{n,A}^{text} f - f$ in sup-norm, when $(a_{n,k})$ is a monotonic sequences with respect to k. Many years later, L. Leindler [6] replaced the monotonic sequences with rest bounded variation sequences. More general results were obtained by B. Wei and D. Yu in [7] and Xh. Z. Krasniqi in [3], [4] and [5].

In our theorems and corollaries we will formulate the very general conditions for the method of summability, with application of r-th differences of higher orders, and for the moduli of continuity obtaining the best degrees of approximation.

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The Toeplitzness of weighted composition operators on Banach spaces of holomorphic functions

Paweł Mleczko CT

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During the talk I will discuss the concepts of various kinds of asymptotically Toeplitz operators acting on Banach spaces of holomorphic functions, including Hardy spaces, Hardy–Lorentz spaces, and Hardy–Orlicz spaces. I will pay special attention to weighted composition operators. In particular I will give a function-theoretic characterisation of uniformly asymptotically Toeplitz and mean weakly asymptotically Toeplitz weighted composition operators on Banach spaces of holomorphic functions.

On multilinear extension of weakly mid p-summing operators

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CT

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In this paper, we introduce and study a new and improved multilinear generalization for the class of weakly mid p-summing operators. This multi-ideal is obtained by combining some known notions of non-linear summability from absolutely p-summing operators with weakly mid-p-summing operators. We refer to this multi-ideal as the class of factorable strongly mid p-summing multilinear operators, owing to its close connection with the Banach multi-ideal of factorable strongly p-summing operators. We prove that, this multilinear generalization best inherits the spirit of weakly mid p-summing operators from the linear theory. We also establish its relation with the previous multilinear generalization, the class of weakly mid-$(p_1, p_2, ..., p_m)$-summing multilinear operators.

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Joint backward extensions of weighted shifts on directed trees

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Weighted shifts on ℓ^2 are well known in operator theory and serve as important class of examples. In [1], the concept was generalised to *weighted shifts on directed trees*, where we replace the linear order of coordinates by a more involved graph structure. The study of backward extension is about whether a given family of weighted shifts on directed trees can be extended to a weighted shift on a single tree, preserving some operator-theoretic property. Among the considered properties are subnormality, power hyponormality and complete hyperexpansivity. In the case of the first two classes the existence of joint backward extension for a family does not depend neither on the additional structure of an enveloping tree nor on any interrelation between the given operators.

The results come from my PhD dissertation written under the supervision of prof. Jan Stochel and were also published in [2,3].

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List of Abstracts – Talks

Amenability in Orlicz sequence algebras. A Hudzik–Domański Handshake.

Krzysztof Piszczek CT

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The notion of amenability arose in early 40's of the XX century. Initially in the context of countable (and quickly after locally compact) groups and later on transferred into the framework of Banach algebras (the breakthrough result of Barry Johnson). One may think of amenable groups as those admitting no paradoxical decompositions (the famous Banach–Tarski Paradox which we will start from). The notion of amenability reveals many deep connections between locally compact groups on the one hand and group algebras on the other. We will briefly highlight these relations together with some historical notes. The main part of the talk will be devoted to amenability properties in Orlicz Sequence Algebras. We will show that also in this framework the condition $\Delta_2(0)$ is crucial for our study.

The talk is based on a joint work with Paweł Foralewski.

Interpolation theory of Down spaces over general measure spaces

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We consider a measure space together with a totally ordered subset of its sigma algebra called an ordered core. Recently, this construction was used in the context of Hardy inequalities, giving a uniform treatment of many different types of Hardy operators.

We will begin by introducing a definition of monotone functions compatible with the ordered core. This allows us to extend the down space construction, a variant of the Köthe dual restricted to positive decreasing functions, to all measure spaces. We will look at their associate spaces and their relationship with a suitable version of the least decreasing majorant construction in this more general setting. We will discuss the interpolation structure of these spaces and find strong similarities to the real line case; the down spaces corresponding to L^1 and L^∞ form an exact Calderón couple and as a consequence, we can describe all their exact interpolation spaces in terms of the $K-$functional. We will also show an analogous result for the dual couple.

This talk is based on joint work with Gord Sinnamon in [1].

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Self-improving boundedness of the Hardy-Littlewood maximal operator over spaces of homogeneous type

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A space of homogeneous type (Ω, d, μ) is a quasi-metric space (Ω, d) with a doubling Borel measure μ—and as a far-reaching generalization of the Euclidean space $\mathbb{R}^n$ with the Lebesgue measure, it constitutes a rich in tools setting for analysis. One may find an analogue of the Euclidean dyadic cubes there [1]; consider the Hardy-Littlewood operator

$$Mf(x) := \sup_{B \ni x} \frac{1}{\mu(B)} \int_B |f(y)| d\mu(y), \quad x \in \Omega,$$

perfectly defined due to the notion of a quasi-distance d and the associated balls B; study the boundedness of M on various function spaces over spaces of homogeneous type, using the dyadic technique. In this work, for quasi-Banach function lattices $X(\Omega, d, \mu)$ with the Fatou property, we generalize the result of Lerner and Ombrosi [2] about self-improvement properties of the maximal operator on quasi-Banach function spaces over $\mathbb{R}^n$. Namely, addressing multiple convexifications of the lattice $X(\Omega, d, \mu)$ defined as

$$X^{(r)}(\Omega, d, \mu) := \{\text{measurable } f \ : \ |f|^r \in X(\Omega, d, \mu)\}, \quad r > 0,$$

we prove that the following are equivalent:

(i) *M is bounded on $X(\Omega, d, \mu)$.*

(ii) *For all $s > 1$, M is bounded on $X^{(s)}(\Omega, d, \mu)$ and*

$$\lim_{s \to 1^+} (s-1) \|M\|_{X^{(s)} \to X^{(s)}} = 0.$$

(iii) *There exists $r_0 \in (0,1)$ such that if $r \in (r_0, 1)$, then M is bounded on $X^{(r)}(\Omega, d, \mu)$.*

An important theoretic application of this result is the special case when $X(\Omega, d, \mu)$ is the variable Lebesgue space $L^{p(\cdot)}(\Omega, d, \mu)$.

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Bochner representable operators on variable exponent Lebesgue spaces

Juliusz Stochmal CT

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Let $L^{p(\cdot)}(\Omega)$ denote a variable exponent Lebesgue space and X be a Banach space. We discuss when a bounded linear operator $T : L^{p(\cdot)}(\Omega) \to X$ is Bochner representable, that is, there exists a unique function $g \in L^{p^*(\cdot)}(\Omega, X)$ such that

$$T(u) = \int_{\Omega} u(t)g(t)\,d\mu, \quad \text{for all } \ u \in L^{p(\cdot)}(\Omega),$$

where $\frac{1}{p(t)} + \frac{1}{p^*(t)} = 1$ μ-a.e. As an application, we study the compactness property of these operators.

List of Abstracts – Talks

Uniform convergence of trigonometric series with p-bounded variation coefficients

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It is well-known that there is a great number of interesting results in Fourier analysis established by assuming monotonicity of coefficients. The following classical convergence result can be found in many monographs (see for example [1] and [10]).

Theorem. *Suppose that $b_n \geq b_{n+1}$ and $b_n \to 0$. Then a necessary and sufficient condition for the uniform convergence of the series*

$$\sum_{n=1}^{\infty} b_n \sin nk$$

is $nb_n \to 0$.

This result has been generalized by weakening the monotone condition of the coefficient sequences (for example in [4], [6], [7], [8] and [9]). In this talk we will present some properties of the following class of sequences introduced in [2]:

Definition. *Let $\beta := (\beta_n)$ be a nonnegative sequence, r a natural number and p a positive real number. The sequence of complex numbers $a := (a_n)$ is said to be $(p, \beta, r) -$ general monotone, or $a \in GM\,(p, \beta, r)$, if the relation*

$$(\sum_{n=m}^{2m-1} |\Delta_r a_n|^p)^{\frac{1}{p}} \leq C\beta_m$$

holds for all $m \in \mathbb{N}$, where $C = C\,(a)$ indicates a constant depending only on a and $\Delta_r a_n = a_n - a_{n+r}$.

This class is the generalization of the class considered by E. Liflyand, S. Tikhonov in [5] and B. Szal in [6]. Moreover, we will also give sufficient and necessary conditions for uniform convergence of trigonometric series with $(p, \beta, r) -$ general monotone coefficients.

The talk is based on the joint papers with M. Kubiak [2] and [3].

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Daugavet and diameter two properties in Orlicz-Lorentz spaces

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In this talk, we consider the diameter two properties (D2Ps), the diametral diameter two properties (diametral D2Ps), and the Daugavet property in Orlicz-Lorentz spaces equipped with the Luxemburg norm. First, we characterize the Radon-Nikodým property of Orlicz-Lorentz spaces in full generality by considering all finite real-valued Orlicz functions. To show this, the fundamental functions of their Köthe dual spaces defined by extended real-valued Orlicz functions are computed. If an Orlicz function does not satisfy the appropriate Δ_2-condition, the Orlicz-Lorentz space and its order-continuous subspace have the strong diameter two property. Consequently, given that an Orlicz function is an N-function at infinity, the same condition characterizes the diameter two properties of Orlicz-Lorentz spaces as well as the octahedralities of their Köthe dual spaces. The Orlicz-Lorentz function spaces with the Daugavet property and the diametral D2Ps are isometrically isomorphic to L_1 when the weight function is regular. In the process, we observe that every locally uniformly nonsquare point is not a Δ-point. This fact provides another class of real Banach spaces without Δ-points. This is a joint work with Anna Kamińska and Han Ju Lee.

Discrete Wiener-Hopf sperators on Orlicz sequence spaces

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One of the main questions one can ask about a bounded linear operator on a Banach space is whether the operator is invertible. Sometimes this question can be very difficult to answer. A weaker form of this question is whether the operator is invertible modulo the ideal of all compact operators. Operators with this property are called Fredholm operators.

The aim of this work is to develop the Fredholm theory of discrete Wiener-Hopf operators with symbols which are periodic distributions. We concentrate on discrete Wiener-hopf operators acting on the Orlicz sequence spaces [2], which are far reaching generalizations of the Lebesgue sequence spaces. We try to extend the results for classical ℓ^p spaces [1] to the setting of Orlicz sequence spaces.

We extend the classical Brown-Halmos, Hartman-Wintner and Coburn theorems for discrete Wiener-Hopf operators to the setting of Orlicz spaces. Further, we give necessary and sufficient conditions for their Fredholmness if the symbols are continuous multipliers or, more generally, belong to the Douglas-type algebra $C_\Phi + \overline{H^\infty_\Phi}$.

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List of Abstracts – Talks

On Calderón–Mityagin couples of Tandori spaces

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Recall that the fundamental problem of the interpolation theory ask if, for given a couple of Banach spaces (X_0, X_1), we have concrete description of all interpolation spaces with respect to this couple. For the couple (L_1, L_∞) this problem was solved by Calderón [1] who showed that every interpolation functor $\mathcal{F}$ is equivalent to the real K-method of interpolation on (L_1, L_∞), that is, we can find parameter $\Phi_{\mathcal{F}}$ for which

$$\mathcal{F}(L_1, L_\infty) = (L_1, L_\infty)^K_{\Phi_{\mathcal{F}}}.$$

All Banach couples (X_0, X_1) for which a similar characterization holds are usually called the **Calderón–Mityagin couples**.

For a function f define non-increasing majorant $\widetilde{f}$ by $\widetilde{f}(t) := \operatorname{esssup}_{s \geq t} |f(s)|$. By the **Tandori space** $\widetilde{X}$, given a Banach function space X, we understand the optimal domain for sublinear operator

$$f \rightsquigarrow \widetilde{f},$$

that is, as the largest (in the sense of inclusion) Banach function space with the property that this operator is bounded when acting into X. Explicitly, the space $\widetilde{X}$ is the collection of all measurable functions f such that the norm $\|f\|_{\widetilde{X}} := \left\|\widetilde{f}\right\|_X$ is finite.

The main goal of this talk is to show that the couple of Tandori spaces $(\widetilde{X_0}, \widetilde{X_1})$ form a Calderón–Mityagin couple if, and only if, the corresponding couple of spaces $(X_0^\bigstar, X_1^\bigstar)$ also form a Calderón–Mityagin couple, where X_0, X_1 are either rearrangement invariant spaces or arbitrary function spaces with non-trivial Boyd indices. Here, the symbol $X^\bigstar$ stands for the symmetrization of X. In order to prove this theorem we represent the Tandori space $\widetilde{X_0}$ as a direct sum of L_∞'s with respect to the dyadic discretization of X. This representation is a vast generalization of Grosse-Erdmann's *blocking technique* for L_p spaces (see [2]). Furthermore, we will use these methods to show new couples that are not Calderón–Mityagin couples.

The talk is based on joint work [3] with Tomasz Kiwerski from Poznań University of Technology.

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Fredholm criteria for the algebra generated by Wiener-Hopf operators with continuous symbols on reflexive Orlicz spaces

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We extend the Fredholm criteria for Wiener-Hopf operators with continuous symbols on the Lebesgue space $L^p(\mathbb{R}_+)$, $p \in (1, \infty)$, obtained by Roland Duduchava in the late 1970s, to the setting of reflexive Orlicz spaces $L^\Phi(\mathbb{R}_+)$. We later combine this result with some techniques from the theory of commutative Banach algebras to derive a general Fredholm criteria for any operator T lying in the closed subalgebra generated by all Wiener-Hopf operators with continuous symbols.

List of Participants

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Adam Bohonos	Szczecin, Poland
Maciej Burnecki	Wrocław, Poland
Diana Caponetti	Palermo, Italy
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Małgorzata Czerwińska	Jacksonville, USA
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Beata Deręgowska	Kraków, Poland
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Krzysztof Koczorowski	Poznań, Poland
Paweł Kolwicz	Poznań, Poland
Ryszard Paweł Kostecki	Gdańsk, Poland
Wojciech Kowalewski	Poznań, Poland
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Karol Leśnik	Poznań, Poland
Barbara Lewandowska	Kraków, Poland
Grzegorz Lewicki	Kraków, Poland
Xiaoyan Li	China
Zhen Liu	Jena, Germany
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Alexander Meskhi	Kutaisi, Georgia
Artur Michalak	Poznań, Poland

List of Participants

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Marian Nowak	Zielona G ora, Poland
Aleena Philip	Hyderabad, India
Piotr Pikul	Kraków, Poland
Krzysztof Piszczek	Poznań, Poland
Ryszard Płuciennik	Poznań, Poland
Enrique A. Sánchez Pérez	València, Spain
Michael Prophet	Cedar Falls, USA
Alejandro Santacruz Hidalgo	London, Canada
Alina Shalukhina	Lisbon, Portugal
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Leszek Skrzypczak	Poznań, Poland
Juliusz Stochmal	Bydgoszcz, Poland
Bogdan Szal	Zielona Góra, Poland
Hyung-Joon Tag	Seoul, Republic of Korea
Sandra Mary Thampi	Lisbon, Portugal
Jakub Tomaszewski	Poznań, Poland
Márcio Valente	Lisbon, Portugal
Baoxiang Wang	Xiamen, China
Marek Wisła	Poznań, Poland
Witold Wnuk	Poznań, Poland
Paweł Wójcik	Kraków, Poland

Useful Information

Travel information

Poznań has an airport and direct fast train connections to Berlin, Warsaw and other big cities. The main railway station is located in the center of the city with a direct tram connection to the Faculty of Mathematics and Computer Science of A. Mickiewicz University (tram lines 14 and 97). Using the same trams lines you can get student hostels Zbyszko i Jagienka. Hotel Mercure is located in walking distance from the railway station.

Poznań Airport Ławica is located not far from the center of the city. You can reach the center by taxi or by bus (line 159)

The Faculty of Mathematics and Computer Science, where the conference will take place, can be reached by tram (lines 14,15,16 and 97).

About the faculty

As a unit of a research centre, the Faculty of Mathematics and Computer Science of Adam Mickiewicz University in Poznan has continued an over 100 years old tradition of mathematics in Poznan. It is also one of the best academic centres in the field of computer science in Poland. At present the faculty offers study programmes in four fields: mathematics, computer science, data analysis and data processing, teaching mathematics and computer science. The latter one is unique for the whole country. The faculty also offers postgraduate study programmes.

Conference lectures will be held in Aula A, which can be accessed from the main hall of the Faculty. Aula A is equipped with boards, a projector, an overhead projector and sound equipment. *Coffee breaks* will be served in the main hall next to Aula A. Lunch will be served on level -1, close to the hall.

Wi-Fi will be available during the conference. The Faculty also will provide access to the Eduroam network.

On Wednesday, July 10, the organizers invite you to visit the ICHOT Gate of Poznań complex, located at Gdańska 2 street. There is a multimedia exhibition (available in many languages) devoted to the past and present of Ostrów Tumski in Poznań.

Then, a *conference dinner* will take place at the "Tumska" restaurant located at ul. Ostrów Tumski 5a, Poznan, Poland.

Sponsors